Superfractals among classical self-similar sets

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A fuchsia for Proffessor Misiurewicz

Metric and topological fractals

Definition

A pair $(X, \mathcal{F})$ is called a **fractal** (an **attractor**) if
X - compact space
$\mathcal{F}$ - finite family of continuous selfmaps on X
$X = \bigcup_{f \in \mathcal{F}} f(X)$

For compact $B \subset X$

$$\mathcal{F}(B) = \bigcup_{f \in \mathcal{F}} f(B)$$

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$(X, \mathcal{F})$ - **metric fractal** (IFS-attractor)
X - metric space
$\mathrm{Lip} f < 1$ for every $f \in \mathcal{F}$

$(X, \mathcal{F})$ - **topological fractal** (TIFS-attractor)
X - Hausdorff space
$\mathcal{F}$ - *topologically contracting system*
[for every open cover $\mathcal{U}$ of X there is $m \in \mathbb{N}$ such that for any maps $f_1, \ldots, f_m \in \mathcal{F}$ the set $f_1 \circ \cdots \circ f_m(X) \subset U \in \mathcal{U}$]

Simple observations

Observation 1

$(X, \mathcal{F})$ is a metric fractal $\Rightarrow$ $(X, \mathcal{F})$ is a topological fractal

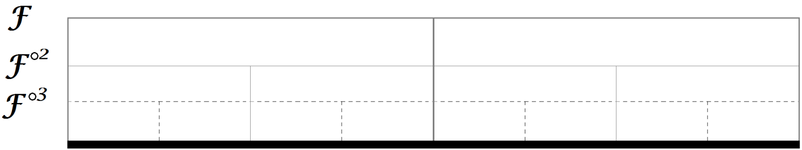
Observation 2

$(X, \mathcal{F})$ is a metric/topological fractal

$\Downarrow$

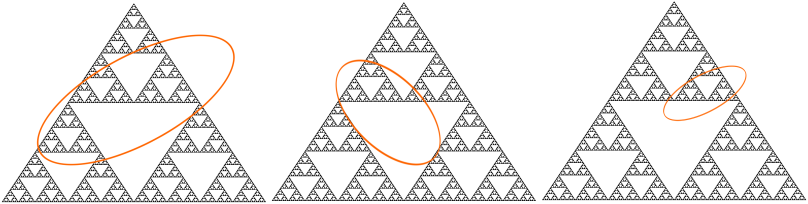
For natural $n > 1$, $(X, \mathcal{F}^{\circ n})$ is a metric/topological fractal

where $\mathcal{F}^{\circ n} = \{f_1 \circ \cdots \circ f_n; f_1, \ldots, f_n \in \mathcal{F}\}$



Definition

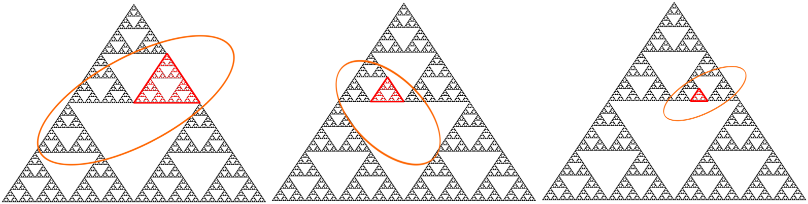
A Hausdorff topological space X is called a **topological superfractal** if for every nonempty, open subset $U \subset X$, $(X, \mathcal{F})$ is a topological fractal for $\mathcal{F}$ such that $f|_{X \setminus U} = const$ for every $f \in \mathcal{F}$.



Brick

Definition

A pair $(B, \mathcal{P})$ is called a **brick** of X if
B is a subset of a topological space X
$\mathcal{P}$ is a finite family of continuous maps $B \to \overline{X \setminus B}$ such that
$\bigcup_{f \in \mathcal{P}} f(B) = \overline{X \setminus B}$.
Moreover if $(B, \mathcal{F})$ is a metric fractal for some $\mathcal{F}$ then $(B, \mathcal{P}, \mathcal{F})$ is called **self-similar brick**.



Main theorem

Theorem

X - compact metric space with self-similar brick $(B, \mathcal{P}, \mathcal{F})$. Then

(***) $\exists \varphi \colon X \to B$ **Lip. on** B cont. surjection, $\varphi|_{\overline{X \setminus B}} = const$

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(*1) $\exists \mathcal{P}'$ finite family of continuous maps $X \to \overline{X \setminus B}$ s.t. $\bigcup_{f\in\mathcal{P}'} f(B) = \overline{X \setminus B}$ and $f|_{\overline{X\setminus B}} = const$ for every $f \in \mathcal{P}'$

(*2) $\exists \mathcal{F}'$ finite family of **Lipschitz on** B, continuous $X \to B$ s.t.

a) $\bigcup_{f\in\mathcal{F}'} f(X) = B$
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X is a topological fractal for family with maps s.t. $f|_{\overline{X \setminus B}} = const$

Proof

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(**) $\forall f \in \mathcal{F}, \exists \varphi_f\colon X \to f(B)$ Lip. on B, cont. sur. $\varphi_f|_{\overline{X\setminus B}} = const$

For $f \in \mathcal{F} = \{g\colon B \to X; \mathrm{Lip} g < 1\}$ take $\varphi_f := f \circ \varphi$.

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$\mathcal{F}' = \{g \circ \varphi_f : X \to B; g \in \mathcal{F}^{\circ k}; f \in \mathcal{F}\}$

$\mathcal{P}' = \{g \circ \varphi_f : X \to \overline{X \setminus B}; g \in \mathcal{P}; f \in \mathcal{F}\}$

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X is a topological fractal for topologically contracting system with maps s.t. $f|_{\overline{X \setminus B}} = const$

$\mathcal{F}' \cup \mathcal{P}'$ is a topologically contractive system on X with maps s.t.

- $f|_{\overline{X \setminus B}} = const$
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Then for every $f_1, \ldots, f_{m+2} \in \mathcal{F}' \cup \mathcal{P}'$ the diameter of image $f_1 \circ f_2 \circ \cdots \circ f_{m+2}(X)$ is $< \varepsilon$

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Corollary

If for compact metric space X for every open subset U there exists a $B \subset U$ self-similar brick satisfies (***) or (**) or (*1,*2) then X is a topological superfractal.

$$f|_{\overline{X \setminus B}} = const \quad \Rightarrow \quad f|_{\overline{X \setminus U}} = const$$

Classical planar fractals are topological superfractals

(***) $\exists \varphi \colon X \to B$ Lipschitz on B, cont. surjection, $\varphi|_{\overline{X \setminus B}} = const$

1. Cantor set
2. Koch curve
3. Sierpiński triangle
4. Sierpiński carpet

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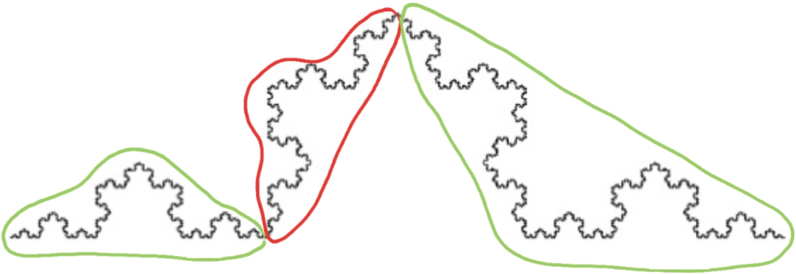

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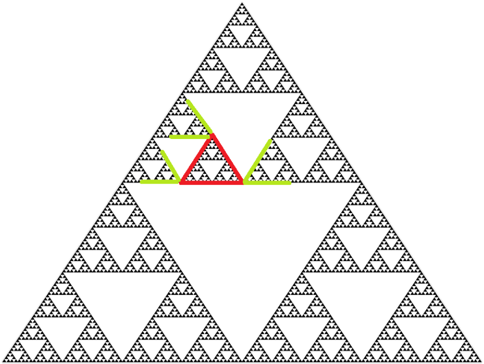

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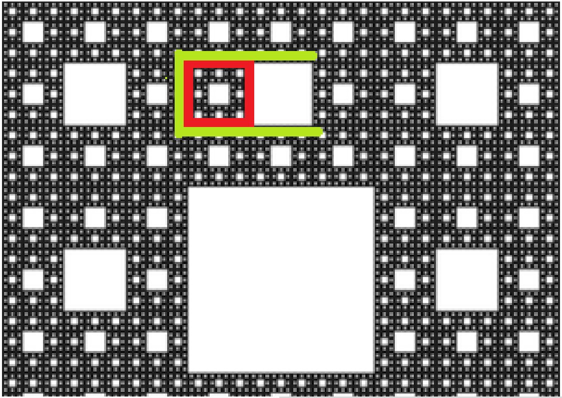

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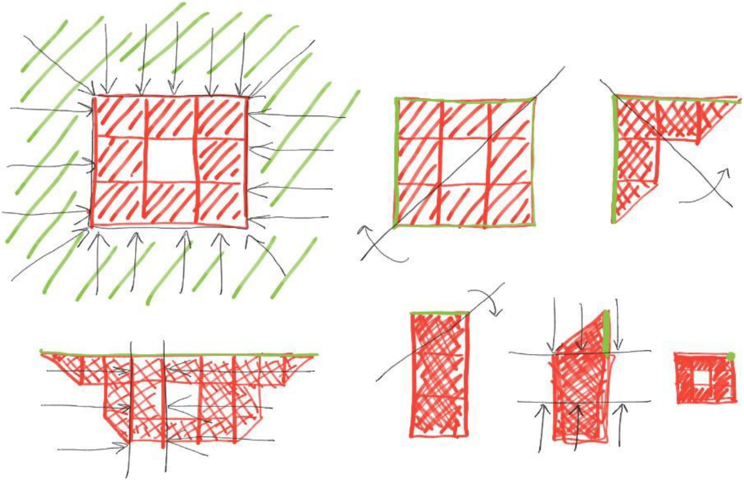

Peano continuum with free arc is a topological fractal

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1. Peano continuum

What is not a topological superfractal

1. $X \cup Y$ where X, Y are disjoint sets with respectively finite and infinite cardinality
   $[0,1] \cup \{2\}$, $\{\frac{1}{n}\}_{n \in \mathbb{N}}$

What is not a topological superfractal

1. $X \cup Y$ where X, Y are disjoint sets with respectively finite and infinite cardinality
   $[0,1] \cup \{2\}$, $\{\frac{1}{n}\}_{n\in\mathbb{N}}$
2. $X \cup Y$ where X, Y are disjoint sets with respectively finite and infinite connected components
   $I \cup \mathcal{C}$, $\{\frac{1}{n}\}_{n\in\mathbb{N}}$

Open problems

Problems

What is the role of

- Open Set Condition
- Lipschitz extensors

for being a topological superfractal?

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$[0,1] \cup \{2\}$ - has OSC but is not a top. superfractal

THANK YOU