Conference on Dynamical Systems
Celebrating
Michał Misiurewicz's 70th Birthday

Jagiellonian University and AGH University of Science and Technology
June 10-14, 2019, Kraków, Poland

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ABSTRACTS

Coding of geodesic flow from Fuchsian groups

Adam Abrams
Polish Academy of Science, Poland
(joint work with **Svetlana Katok**)

We describe several related ways in which geodesic flow on a compact surface of constant negative curvature and genus $g \geq 2$ can be realized as a special flow over a symbolic system. This is intuitively related to the coding of geodesic flow on the modular surface by continued fractions, with $PSL(2, \mathbb{Z})$ now replaced by a Fuchsian "surface group". Katok and Ugarcovici describe an $(8g-4)$-parameter family of maps generalizing the Bowen–Series boundary map. For various parameter choices, the natural extension of the boundary map has a global attractor with finite rectangular structure, this attractor parameterizes a cross-section for the geodesic flow, and the first return map to this cross-section is a shift map. In special cases where the "cycle ends" are discontinuity points of the boundary map, the resulting symbolic system is sofic. Recent results focus on the boundary of the parameter space.

References

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New results on the combinatorial dynamics of the minimum entropy degree one circle maps depending on the rotation interval and its use as examples factory for graph maps

Lluís Alsedà
Autonomous University of Barcelona, Spain
(joint work with **Jorge Groisman** and **Liane Bordignon**)

The minimum entropy degree one circle maps depending on the rotation interval are known since 1988 [1]. In this talk I will obtain the intertwinning of the the two extremal twist orbits of these maps when the endpoints of the interval are rational and I will show that this information characterizes completely the dynamics of such maps. In a second part (time permitting) I will show how to extend these maps to an arbitrary graph with a single circuit by essentially keeping the dynamics. This is a factory of particular examples with (a limited) dynamics, analogous to circle dynamics of the the minimum entropy degree one circle maps.

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Zigzags in interval inverse limits

Ana Anušić
University of São Paulo, Brazil
(joint work with **Henk Bruin** and **Jernej Činč**)

We will discuss how the presence of *zigzags* in bonding maps affects the topology of the inverse limit. For example, zigzags in bonding maps of X will often present an obstacle to constructing planar embeddings with certain $x \in X$ accessible. Moreover, zigzags will often produce double spirals or nasty points, i.e. points not contained in any arc of the space (e.g. points in the pseudo-arc). Every iterate of a unimodal map is zigzag-free, implying some interesting topological consequences. For example, one can show that for every point x in unimodal inverse limit (UIL) X there exists an embedding of X in the plane under which x is accessible. Moreover, one can characterize endpoints of UILs as points which are endpoints of their basic arcs, or as points which are endpoints of every arc which contains them.

Bracket polynomials, uniform distribution and dynamics

Vitaly Bergelson
Ohio State University, United States

A classical theorem due to H. Weyl states that if P is a polynomial over $\mathbb{R}$ such that at least one of its coefficients (other than the constant term) is irrational, then the sequence $P(n)$, $n = 1, 2, \dots$ is uniformly distributed mod 1. We will discuss some extensions of this theorem which involve "bracket polynomials" (also called "generalized polynomials"), that is, functions which are obtained from the conventional polynomials by the use of the greatest integer function, addition and multiplication. We will also explain the intrinsic connections between the generalized polynomials, dynamical systems on nil-manifolds, and polynomial extensions of Szemeredi theorem on arithmetic progressions.

Topological entropy, upper capacity and fractal dimensions of finitely generated semigroup actions

Andrzej Biś
University of Łódź, Poland
(joint work with **Dikran Dikranjan, Anna Giordano Bruno** and **Luchezar Stoyanov**)

We show that the topological entropy of a finitely generated semigroup $\{G, G_1\}$ of continuous sef-maps of a compact metric space X coincides with the limits of upper capacities of dynamically defined Caratheodory structures on X depending on the generating set G_1. Moreover, the topological entropy of a semigroup is lower estimated by the generalization of Katok's delta-measure entropy. For locally L-expanding semigroup $\{G, G_1\}$ its topological entropy dominates the product the Hausdorff dimension of X and logarithm of L.

On the entropy conjecture of Marcy Barge

Jan P. Boroński
AGH University of Science and Technology, Poland
(joint work with **Jernej Činč** and **Piotr Oprocha**)

I shall discuss a positive solution to the following problem, obtained in a joint work with J. Činč and P. Oprocha.

Question (M. Barge, 1989 [8]) Does there exist, for every $r \in [0, \infty]$, a pseudo-arc homeomorphism whose topological entropy is r?

Until now all known pseudo-arc homeomorphisms have had entropy 0 or ∞. Recall that the pseudo-arc is a compact and connected space (continuum) first constructed by Knaster in 1922 [6]. It can be seen as a pathological fractal. According to the most recent characterization [5] it is topologically the only, other than the arc, continuum in the plane homeomorphic to each of its proper subcontinua. The pseudo-arc is homogeneous [2] and played a crucial role in the classification of homogeneous planar compacta [4]. Lewis showed that for any n the pseudo-arc admits a period n homeomorphism that extends to a rotation of the plane, and that any P-adic Cantor group action acts effectively on the pseudo-arc [7] (see also [10]). We adapt Lewis' inverse limit constructions, by combining them with a Denjoy-Rees scheme [1] (see also [9, 3]). The positive entropy homeomorphisms that we obtain are periodic point free, except for a unique fixed point.

I shall start my talk by reviewing the role that the pseudo-arc have played in various areas of mathematics, including topology, surface dynamics, complex analysis, isometric theory of Banach spaces and logic, and then will move on to to the history of the problem, to conclude with a discussion of its solution.

References

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Self-similarity of the entropy curve for "matching" interval maps

Henk Bruin
University of Vienna, Austria
(joint work with **Carlo Carminati, Alessandro Profetti** and **Stefano Marmi**)

Entropy is a way of quantifying the amount of chaos in a dynamical system can exhibit, and for iterated maps on the interval it can be computed with great precision. In general, entropy can vary quite irregularly as function of the parameters of the system. It is therefore curious to observe for certain families of interval maps with discontinuities that are "resolved" by some matching procedure, that the entropy function exhibits a type of self-similarity. In this talk I want to present this phenomenon, and explanations behind it. Similar phenomena were seen in e.g. work of Dajani and Kalle.

Dynamics on algebraic groups in positive characteristic

Jakub Byszewski
Jagiellonian University, Poland
(joint work with **Gunther Cornelissen, Marc Houben** and **van der Meijden**)

We study periodic points of endomorphisms of algebraic groups over fields of positive characteristic p as well as (compactifications) of their quotients by finite group actions (the so-called dynamically affine maps). This is a positive characteristic analogue of many classical maps: toral endomorphisms, Chebyshev and Lattes maps, and endomorphisms of semisimple algebraic groups as studied by Steinberg.

Periodic points are counted by the dynamical zeta function of Artin and Mazur. We study its rationality and the length distribution of orbits, obtaining a rational/transcendental dichotomy depending on arithmetic properties of the map.

Folding points and endpoints

Jernej Činč
AGH University of Science and Technology, Poland
& IT4Innovations, Czech Republic
(joint work with **Lori Alvin, Ana Anušić** and **Henk Bruin**)

In this talk I will review some recently obtained [1] results and questions which concern inhomogenaities of unimodal inverse limit spaces (UIL's). Among other results we will state a characterization of UIL's for which the set of folding points (points with no neighbourhood being Cantor set of open arcs) equals the set of its endpoints. Moreover, we will also discuss how the last result applies for more general classes of one-dimensional continua.

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Ergodicity of systems of interval homeomorphisms with place-dependent probabilities

Klaudiusz Czudek
Polish Academy of Science, Poland

Markov chains generated by iterated function systems consisting of homeomorphisms of the interval gained a lot of interest during last years. It was started by the paper of L. Alsedá and M. Misiurewicz [1], where the authors proved the ergodicity of the Markov chain, provided that the system consists of two homeomorphisms of very specific form, which are chosen each with probability $1/2$. This result was later generalized in [2] to the case of two arbitrary orientation preserving diffeomorphisms chosen with arbitrary probabilities such that Lyapunov exponents on the boundary are positive. Further, T. Szarek and A. Zdunik generalized this result to the case of arbitrary number of orientation preserving homeomorphisms of the interval. The Central Limit Theorem for this type of Markov chains was established recently by T. Szarek and myself. I would like to present a sketch of the proof of ergodicity of the Markov chain in the case when probabilities are place-dependent and homeomorphisms are of the specific form from the origin paper [1]. Surprisingly, we do not need to assume, that the system is contractive in average, which was a crucial assumption in previous criteria for ergodicity of Markov chains arising from iterated function systems.

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Knobbly but nice

Neil Dobbs
University College Dublin, Ireland

Our main result states that, under an exponential map whose Julia set is the whole complex plane, on each piecewise smooth Jordan curve there is a point whose orbit is dense. This has consequences for the boundaries of *nice sets*, used in induction methods to study ergodic and geometric properties of the dynamics.

In 1997 Kamae proved that an infinite subset $A = \{n_k\}$ of natural numbers preserves normality (that is for each normal sequence (x_n) the subsequence (x_{n_k}) is also normal) if and only if $\{n_k\}$ is deterministic in the sense of Weiss. In my talk I will discuss how this result can be generalized to all countable amenbale groups.

Normality preserving subsets of countable amenable groups

Tomasz Downarowicz
Wrocław University of Technology, Poland
(joint work with **Vitaly Bergelson** and **Joseph Vandehey**)

In 1997 Kamae proved that an infinite subset $A = \{n_k\}$ of natural numbers preserves normality (that is for each normal sequence (x_n) the subsequence (x_{n_k}) is also normal) if and only if $\{n_k\}$ is deterministic in the sense of Weiss. In my talk I will discuss how this result can be generalized to all countable amenbale groups.

Nonexpansiveness in Z^2 symbolic dynamics

John Franks
Northwestern University, United States

We consider Z^2-subshifts, i.e. closed Z^2 invariant subsets of A^{Z^2} where A is a finite alphabet. Our focus is on how the geometry of nonexpansive subspaces controls the propagation of information in such systems.

The bifurcation set as a topological invariant for one-dimensional dynamics

Gabriel Fuhrmann
Imperial College London, United Kingdom
(joint work with **Maik Gröger** and **Alejandro Passeggi**)

For a continuous map on the unit interval or circle, we define the bifurcation set to be the collection of those interval holes whose surviving set is sensitive to arbitrarily small changes of their position. By assuming a global perspective and focusing on the geometric and topological properties of this collection rather than the surviving sets of individual holes, we obtain a novel topological invariant for one-dimensional dynamics.

We provide a detailed description of this invariant in the realm of transitive maps and observe that it carries fundamental dynamical information. In particular, for transitive non-minimal piecewise monotone maps, the bifurcation set encodes the topological entropy and strongly depends on the behavior of the critical points.

Coarse entropy

William Geller
Indiana University-Purdue University Indianapolis, United States
(joint work with **Michał Misiurewicz**)

We define a large-scale version of topological entropy for maps of unbounded spaces. Its value coincides with that of Rufus Bowen's noncompact entropy for linear maps of Euclidean spaces, for example. However, unlike Bowen's entropy, it is invariant under coarse conjugacies, i.e. quasi-isometries of spaces that almost commute with the dynamics. Thus, for example, its value is log 2 for the doubling map on the integers, which is coarsely conjugate to the doubling map on the reals, whereas Bowen's entropy vanishes for this digitized doubling map. We derive some properties of the coarse entropy and compute it in some relevant examples.

Between complex dynamics and analysis: Painlevé and Bowen problems

Jacek Graczyk
Paris-Sud University, France
(joint work with **Peter Jones** and **Nicolae Mihalache**)

We will discuss how asymptotically sublinear measures of bounded Menger curvature can be useful to study metric properties of sets which are far away from smooth curves at scales of positive density and their relationships with Painlevé problem about removable sets for bounded analytic functions and Bowen's dichotomy for rational maps. The simplest examples of the sets with asymptotically sublinear measures of extremal growth are van-Koch type snowflakes and Misiurewicz Julia sets. Another extremal class is given by generic unicritical Julia sets with respect to harmonic measure on the bifurcation loci.

Topological degree of sphere maps preserving multipole foliations

Grzegorz Graff
Gdańsk University of Technology, Poland
(joint work with **Michał Misiurewicz** and **Piotr Nowak-Przygodzki**)

We consider a continuous self-map f of a two-dimensional sphere $\mathbb{S}^2$ preserving some type of foliations having many singularities. The analyzed cases cover the family of foliations with one singularity studied in [1] (so-called *rabit foliations*). We provide the characterization of the structure of considered foliations, as well as find the restriction for the values of topological degree of f, $\deg(f)$, stating that $|\deg(f)| \leq 1$.

References

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Semiattractors of nonautonomous and random dynamical systems

Grzegorz Guzik
AGH University of Science and Technology, Poland

We summarize the theory of semiattractors of induced semiflows of multifunctions associated with nonautonomous and random dynamical systems given by cocycles mappings. We present sufficient conditions for existence, consider fiber structure and discuss connections between semiattractors and nonautonomous/random point attractors from both topological and measure-theoretical point of view.

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Some comments on the Entropy of Delone sets

Till Hauser
Friedrich Schiller University Jena, Germany

In order to study Delone sets of finite and infinite local complexity in Euclidean space, we present that a version of the Ornstein Weiss Lemma holds for any compactly generated locally compact abelian group. From this, we conclude that patch counting entropy can be calculated as a limit along certain Van Hove sequences.

A construction of Mary Rees

Judy Kennedy
Lamar University, United States
(joint work with **Paula Ivon Vidal Escobar, Jan Boroński, Piotr Oprocha** and **Xiaochuan Liu**)

Around 1980 Mary Rees [4] took a torus homeomorphism and carefully modified it to obtain the properties she desired. That construction (also called the Denjoy-Rees Technique) is quite complicated, but it works. Since then a number of authors have used her construction and generalizations of it to construct other homeomorphisms, including quite recently. See, for example, [1, 2, 3]. We investigate her technique, how and when it works, and how much it can be generalized.

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Substitutive systems and Cobham's theorem

Elżbieta Krawczyk
Jagiellonian University, Poland

A sequence is substitutive if it arises as an image under a coding of a fixed point of a morphism acting on a finite alphabet. If the morphism can be taken to be of constant length k, then the sequence is called k-automatic. Automatic sequences have links with multiple branches of mathematics and computer science: they appear, for example, in formal language theory, number theory (most notably, in transcendence theory) and fractal geometry. One of the most fundamental results about automatic sequences is Cobham's theorem which classifies sequences that are simultaneously automatic with respect to two multiplicatively independent bases: these are precisely the sequences that are ultimately periodic.

Substitutive sequences give rise to a well-known class of dynamical systems, the so-called substitutive systems. Such systems were extensively studied in the context of primitive substitutions, necessarily restricting such studies to minimal systems. During the talk we will focus on non-minimal substitutive systems: we will discuss the dynamical structure of such systems as well as their links with Cobham's theorem. In particular, we will show that all transitive subsystems of substitutive systems are substitutive. As an application we will obtain a finitary version of Cobham's Theorem which gives a complete characterisation of sets of words that can occur as common factors of two multiplicatively independent automatic sequences.

Entropy density of ergodic measures for B-free shifts

Michal Kupsa
Czech Academy of Sciences, Czech Republic
(joint work with **Jakub Konieczny and Dominik Kwietniak**)

We study a class of symbolic dynamical systems which can be approximated in the d-bar pseudometric by sequences of sofic shifts. (The d-bar pseudometric on the shift space equals upper asymptotic density of the set of coordinates where given two points in the shift differ.) This includes the hereditary closure of any shift space generated by the characteristic function of a set of B-free integers (B-free shifts). An integer is B-free if it is not divisible by any integer from a given set B. We also explore other families of shift spaces related to arithmetic progressions, for example rational shifts defined as orbit closures of points in the d-bar closure of finite unions of arithmetic progressions. Our main result states that ergodic measures are entropy-dense among all invariant measures of the hereditary closure of any B-free or rational shift.

Rigidity in negative curvature

François Ledrappier
Sorbonne University, France

I will survey some results and problems related to recognizing compact manifolds with negative curvature that are locally symmetric spaces.

On Möbius disjointness conjecture

Mariusz Lemańczyk
Nicolaus Copernicus University in Toruń, Poland

The talk will be devoted to the recent progress on the Möbius disjointness conjecture of P. Sarnak:

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n\leq N} f(T^n x)\mu(n)=0$$

for each zero entropy dynamical system (X,T), all $f\in C(X)$ and $x\in X$; μ denotes the arithmetic Möbius function.

Entropy on modules over the group ring of a sofic group

Bingbing Liang
Polish Academy of Sciences, Poland

We partially generalize Peters' formula on modules over the group ring $\mathbb{F}G$ for a given finite field $\mathbb{F}$ and a sofic group G. That is, the entropy of a finitely generated group ring module coincides with the sofic topological entropy of the associated algebraic action.

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Continuous orbit equivalence for wild Cantor actions

Olga Lukina
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(joint work with **Steve Hurder**)

The notion of continuous orbit equivalence for group actions on Cantor sets was introduced by Boyle in his thesis [1]. Boyle also proved the first rigidity result in this area, namely, that two actions of the group of integers $\mathbb{Z}$ on a Cantor set C are continuously orbit equivalent if and only if they are flip conjugate.

More recently, continuous orbit equivalence for equicontinuous actions of finitely generated groups on a Cantor set C was studied by Li [6] and by Cortez and Medynets [2]. In both works, an important assumption is that the actions are topologically free, which means that the set of points with trivial stabilizers is dense in C.

In this talk, we concentrate on actions of finitely generated groups on a Cantor set C which are *not* topologically free. Such actions are ubiquitous in mathematics, arising, for example, as actions of iterated monodromy groups in geometric group theory, actions associated to arboreal representations in arithmetic dynamics, and in other contexts. For some such actions, the induced holonomy action on a sufficiently small clopen subset of C is topologically free. We call such actions *stable*. Other actions never stabilize, and we call such actions *wild*.

For actions which are not topologically free, we discuss an appropriate notion of rigidity, and introduce two group-theoretical invariants of continuous orbit equivalence. We investigate these invariants, and their relation to the topology of the associated étale groupoids, for stable and wild actions. We show that these invariants are non-equal if the topology of the groupoid associated to the action is non-Hausdorff.

The talk is based on papers [3, 4, 5].

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Borel complexity of sets of normal numbers via generic points in subshifts with specification

Bill Mance
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We study the Borel complexity of sets of normal numbers in several numeration systems. Taking a dynamical point of view, we offer a unified treatment for continued fraction expansions and base r expansions, and their various generalisations: generalised Lüroth series expansions and β-expansions. In fact, we consider subshifts over a countable alphabet generated by all possible expansions of numbers in $[0, 1)$. Then normal numbers correspond to generic points of shift-invariant measures. It turns out that for these subshifts the set of generic points for a shift-invariant probability measure is precisely at the third level of the Borel hierarchy (it is a $\mathbf{\Pi}^0_3$-complete set, meaning that it is a countable intersection of F_σ-sets, but it is not possible to write it as a countable union of G_δ-sets). We also solve a problem of Sharkovsky–Sivak on the Borel complexity of the basin of statistical attraction. The crucial dynamical feature we need is a feeble form of specification. All expansions named above generate subshifts with this property. Hence the sets of normal numbers under consideration are $\mathbf{\Pi}^0_3$-complete.

Spiders' webs in the punctured plane

David Martí-Pete
Polish Academy of Sciences, Poland
(joint work with **Vasiliki Evdoridou** and **David J. Sixsmith**)

Many authors have studied sets, associated with the dynamics of a transcendental entire function, which have the topological property of being a spider's web. Roughly speaking, a set is a spider's web if it is connected, and it contains a sequence of "loops" surrounding each other and filling the plane. This concept was introduced by Rippon and Stallard, and proved very useful in understanding the structure of the escaping set of transcendental entire functions. In our work [1], we adapt the definition of a spider's web to the punctured plane. We give several characterisations of this topological structure, and study the connection with the usual spider's web in the complex plane. We show that there are many transcendental self-maps of $\mathbb{C}^*$ for which the Julia set is such a spider's web, and we construct the first example of a transcendental self-map of $\mathbb{C}^*$ for which the escaping set is such a spider's web and, in particular, is connected.

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On Hausdorff dimension of thin nonlinear solenoids

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(joint work with **Feliks Przytycki** and **Michał Rams**)

Let $M = S^1 \times \mathbb{D}$ be the solid tours, where $S^1 = \frac{\mathbb{R}}{2\pi\mathbb{Z}}$, $\mathbb{D} = \{v \in \mathbb{R}^2 || v| < 1\}$ carries the product distance $d = d_1 \times d_2$ and suppose $S : M \to M$ such that

$$(x, y, z) \mapsto (\eta(x, y, z) \bmod 2\pi, \lambda(x, y, z), \mu(x, y, z))$$

is a smooth embedding map where η, λ and μ are close to constant.
Bothe [1] was the first who obtained results on the dimension of the attractor of a thin linear solenoid where contraction rates are strong enough. Barriera, Pesin and Schemeling [2] established a dimension product structure of invariant measures in the course of proving the Eckmann Ruelle conjecture.
Conjecture. The fractal dimension of a hyperbolic set is (at least generically or under mild hypotheses) the sum of those of its stable and unstable slices, where fractal can mean either Hausdorff or upper box dimension.
In spite of the difficulties due to possible low regularity of the holonomies, indeed, Hasselblatt and Wilkinson [4] found open sets of symplectic Anosov maps with the property that on a residual set of full measure (with respect to any invariant measure) the subbundles are not Lipschitz, and the holonomies are non-Lipschitz a.e. with respect to Lebesgue measure. Hasselblat and Schmeling [3] have proved the conjecture for a class of thin linear solenoids. We prove the conjecture for a class of thin nonlinear solenoids.

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Superfractals among classical self-similar sets

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Consider the finite family $\mathcal{F}$ of continuous self-maps on the topological space X and the space $\mathcal{H}(X)$ of nonempty, compact subsets of X. We can define a dynamical system $(\mathcal{H}(X), \mathcal{F})$ such that for each $K \in \mathcal{H}(X) \quad \mathcal{F}(K) = \bigcup_{f\in\mathcal{F}} f(K)$.

By the *self-similar set* or *fractal* we understand a nonempty compact set $A \subset X$ such that

$$A = \mathcal{F}(A) = \bigcup_{f\in\mathcal{F}} f(A)$$

and for every compact set $K \in \mathcal{H}(X)$ the sequence $\big(\mathcal{F}^n(K)\big)_{n=1}^{\infty}$ converges to A in the Vietoris topology on $\mathcal{H}(X)$. In the case when X is a complete, metric space and $\mathcal{F}$ contains Banach contractions, the self-similar set is called an *attractor of iterated function system* $\mathcal{F}$ or *IFS-attractor.*

We consider *topologically contracting* family $\mathcal{F}$ on the self-similar set A which means that for every open cover $\mathcal{U}$ of A there is $n \in \mathbb{N}$ such that for any maps $f_1, \dots, f_n \in \mathcal{F}$ the set $f_1 \circ \dots \circ f_n(A)$ is contained in some set $U \in \mathcal{U}$. A Hausforff toplogical space A is defined to be a **topological superfractal** if for every non-epmpty open set $U \subset A$ there is a topologically contracting family $\mathcal{F}$ such that A is a self-similar set for $\mathcal{F}$ and for every $f \in \mathcal{F}$ the restriction $f|(A \setminus U)$ is a constant map.

The notion of superfractal much better reflects the intuitive perception of self-similarity. We present some classical fractals which are superfractals.

Hamiltonian Markov Chains: functional dynamics

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(joint work with **Soumyadib Ghosh** and **Yingdong Lu**)

HMC is an approach to a problem of finding a normalizing constant when a distribution is known only up to a proportionality factor. The normalization becomes crucial when a sampling is needed. For example when a set is given by some constraints a characteristic function represents a uniform distribution on this set up to a constant which is the volume of the set, usually difficult to compute. Several algorithms were developed and HMC is doing very well among them. We provide an explanation of the phenomenon in terms of pure functional-analytical methods.

Nonexistence of wandering domains for infinitely renormalizable Hénon maps with stationary combinatorics

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We generalize the nonexistence of wandering domains from unimodal maps to strongly dissipative infinitely renormalizable Hénon-like maps with arbitrary stationary combinatorics. This solves an open problem proposed by van Strien [1] and Lyubich and Martens [2].

To prove the theorem, we partition the phase space of a Hénon-like map into two regions: the good region and the bad region. The good region is where the method of proof for unimodal maps applies to Hénon-like maps, while the bad region is where serious difficulties occur. These difficulties are resolved by the Two-Row Lemma, an inequality that relates the contraction of areas to the contraction of bad regions. After analyzing the competition of the two types of contraction, we show that the case of bad regions happens at most finitely many times and complete the proof.

As an application, the theorem enriches our understanding of the topological structure of the heteroclinic web: the union of the stable manifolds of periodic orbits forms a dense set in the domain.

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Transitivity and mixing for expanding Lorenz maps

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(joint work with **Piotr Oprocha** and **Paweł Potorski**)

Suppose that $f : [0,1] \to [0,2]$ is a continuous strictly increasing function which is differentiable on $(0,1) \setminus F$ where F is a finite set. Moreover assume that $\beta := \inf_{x \in (0,1)\setminus F} f'(x) > 1$. Hence there exists a unique $c \in (0,1)$ with $f(c) = 1$. Define $T_f x := f(x) - \lfloor f(x) \rfloor$, where $\lfloor y \rfloor$ is the largest integer smaller or equal to y. A map T_f of this form is called an expanding Lorenz map. Note that T_f has a discontinuity at c.

It is investigated when T_f is topologically transitive and topologically mixing in the case $\beta \geq \sqrt[3]{2}$. One obtains that in the case $\beta \geq \sqrt[3]{2}$ and $f(0) \geq \frac{1}{\beta+1}$ the map T_f is topologically transitive. Moreover it is also topologically mixing except in the case $f(x) = \sqrt[3]{2}x + \frac{2+\sqrt[3]{4}-2\sqrt[3]{2}}{2}$ for all $x \in [0,1]$.

For the special case $f(x) = \beta x + \alpha$ better results are obtained. Here one can completely describe the set of all (β, α) with $\sqrt[3]{2} \leq \beta \leq 2$ and $0 \leq \alpha \leq 2 - \beta$ such that T_f is topologically transitive. Except in three special cases all of these topologically transitive maps are also topologically mixing.

Using a definition of Glendinning the map T_f is called locally eventually onto if every nonempty open $U \subseteq [0,1]$ contains open intervals $U_1, U_2 \subseteq U$ and there are $n_1, n_2 \in \mathbb{N}$ such that ${T_f}^{n_1}$ maps U_1 homeomorphically to $(0,c)$ and ${T_f}^{n_2}$ maps U_2 homeomorphically to $(c,1)$. The map T_f is called renormalizable if there are $0 \leq u_1 < c < u_2 \leq 1$ and $l, r \in \mathbb{N}$ with $l + r \geq 3$ such that ${T_f}^l$ is continuous on (u_1, c), ${T_f}^r$ is continuous on (c, u_2), $\lim_{x\to c^-} {T_f}^l x = u_2$ and $\lim_{x \to c^+} {T_f}^r x = u_1$. An example of a renormalizable expanding Lorenz map is given which is also locally eventually onto. Finally a condition closely related to "locally eventually onto" is defined and it is proved that this condition is equivalent to T_f is not renormalizable.

Lyapunov spectrum for affine iterated function systems on the plane

Michał Rams
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(joint work with **Balazs Barany, Thomas Jordan** and **Antti Kaenmaki**)

The affine iterated function system on the plane is a finite family of affine contracting maps in $\mathbb{R}^2$ given as $(f_i)_{i=1}^m; f_i(x) = A_i x + a_i$, where $A_i \in GL(2,\mathbb{R}), ||A_i|| < 1$ and $a_i \in \mathbb{R}^2$. Like every iterated function system, it generates a unique nonempty set Λ satisfying equation $\Lambda = \bigcup_i f_i(\Lambda)$, so-called limit set. For every $\omega \in \Sigma = \{1, \ldots, m\}^{\mathbb{N}}$ we can define $\pi(\omega) = \lim_{n\to\infty} f_{\omega_1} \circ \ldots \circ f_{\omega_n}(0)$, we have $\Lambda = \pi(\Sigma)$. Thus, for $\omega \in \Sigma$ we can define for any $n > 0$ the inverse map $f_\omega^{-n} = f_{\omega_n}^{-1} \circ \ldots \circ f_{\omega_1}^{-1}$ and $f_\omega^{-n}(\pi(\omega)) = \pi(\sigma^n \omega) \in \Lambda$. For a given $\omega \in \Sigma$ we can then define the Lyapunov exponents $\lambda_1(\omega), \lambda_2(\omega)$ by

$$\lambda_i(\omega) = \lim_{n\to\infty} \frac{1}{n} \log \alpha_i(f_\omega^{-n}(\pi(\omega)))$$

(when the limit exists), where $\alpha_i(f)$ is the ith singular value of the linear part of the affine map f. For a given (ℓ_1, ℓ_2) we are interested in the topological entropy of the set of $\omega \in \Sigma$ for which $\lambda_i(\omega) = \ell_i$, this function is called the Lyapunov spectrum.

We say that the strong open set condition is satisfied if $\pi : \Sigma \to \Lambda$ is a bijection. In this talk I will present the formula for the Lyapunov spectrum of the affine iterated function systems satisfying strong irreducibility condition (this is a generic condition on (A_i)) and strong open set condition.

"What's past is prologue"*

Ana Rodrigues
University of Exeter, United Kingdom
(joint work with **Michał Misiurewicz**)

In this talk I will give an overview of my work with Michal that started the day after Serguy Kolyada introduced us.

*William Shakespeare

Constant slope models and perturbation

Samuel Roth
Silesian University in Opava, Czech Republic
(joint work with **Michal Malek**)

We work in the space of transitive, piecewise monotone maps of a fixed modality m with the topology of uniform convergence. There is an operator on this space which assigns to a map its constant slope model. This operator is discontinuous at points (maps) where perturbation can lead to a jump in entropy. Alseda and Misiurewicz conjectured that these are the only discontinuity points. We confirm the conjecture by a technique of "counting preimages."

Fluctuations of ergodic sums on periodic orbits under specification

Samuel Senti
Federal University of Rio de Janeiro, Brazil
(joint work with **Manfred Denker** and **Xuan Zhang**)

We study the fluctuations of ergodic sums by the means of global and local specifications on periodic points and obtain Lindeberg-type central limit theorems. As an application we can prove the weak convergence of ergodic sums to a mixture of normal distributions for systems with a unique measure of maximal entropy. Our results suggest to decompose the variances of ergodic sums according to global and local sources.

Period incrementing and chaos in a hybrid neuron model

Justyna Signerska-Rynkowska
Gdańsk University of Technology, Poland
(joint work with **Jonathan Rubin, Jonathan Touboul** and **Alexandre Vidal**)

Analysis of the complex dynamics arising in single-cell neuron models is one of the objectives in contemporary mathematical neuroscience. The models considered in this talk belong to hybrid dynamical systems, combining continuous dynamics with discrete events and being especially interesting from the mathematical viewpoint. Specifically, we consider a class of (nonlinear) integrate-and-fire (IF) models with ordinary differential equations

$$\begin{cases} \dot{v} = F(v) - w + I \\ \dot{w} = \varepsilon(bv - w), \end{cases} \tag{1}$$

governing evolution of membrane voltage v and adaptation w between consecutive action potentials (spikes), and a reset mechanism

$$v(t) \xrightarrow[t \to t_*]{} \infty \implies \begin{cases} v(t_*) = v_R \\ w(t_*) = \gamma w(t_*^-) + d, \end{cases} \tag{2}$$

imitating the rapid firing of a spike. These systems exhibit a wide range of desirable behaviours and play a prominent role in the field. In particular, previous works suggested numerically the presence of an underlying spike-adding structure ([4]), i.e. incrementing the number of spikes in bursts (groups of spikes fired in a rapid succession alternated by quiescence periods) as reset voltage v_R is progressively increased. Similar scenarios, hard to analyse in detail without extreme timescale separation, were investigated in fundamental works of Rinzel and Troy [Lecture Notes in Biomath., Springer, 1983] on bursting and spike incrementing in the Belousov-Zhabotinsky reaction and Levi [SIAM J. Appl. Math. 1990] on period-adding phenomena observed experimentally in periodically forced neon tubes.

However, for the models (1)-(2) we obtain a fine description of bursting patterns and spike-adding transitions interspersed by regimes of chaotic dynamics by purely analytical methods, often extending beyond the limit of timescale separation ([2]). In our analysis we rely on the fact that the dynamical properties of the IF system are captured by iterations of a one-dimensional *adaptation map*, with fixed points and periodic orbits of this map corresponding, respectively, to regular firing and bursting. Using slow-fast methods with the adaptation being the slow variable, we show that in the limit of timescale separation the adaptation map converges towards a piecewise linear discontinuous map, the orbits of which we exactly characterize. This limit-map shows a period-incrementing structure with instantaneous transitions. Further, we provide a novel rigorous demonstration that the system can support bursts of any period as a function of model parameters and that the period-incrementing cascade persists for non-constant adaptation, but with more complex transitions. Finally, applying the theory of interval maps ([1, 3]) to the arising adaptation map, we characterize the presence of different notions of topological and metric chaos at the transitions.

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Singular stationary measures for Alsedà-Misiurewicz systems

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(joint work with **Krzysztof Barański**)

We consider a family of random dynamical systems, each consisting of two piecewise affine increasing homeomorphisms f_-, f_+ of the unit interval, each with exactly one point of non-differentiability, iterated randomly with probabilities (p_1, p_2). Since systems of this type were introduced in [1] by Alsedà and Misiurewicz, we call them Alsedà-Misiurewicz systems, or AM-systems. Under certain assumptions, such a system admits a unique stationary probability measure μ with no atoms at the endpoints. In this case, μ has to be either singular or absolutely continuous with respect to the Lebesgue measure. We prove that μ is singular for a certain open set of parameters, verifying a conjecture by Alsedà and Misiurewicz [1] in this case. We also prove singularity and calculate Hausdorff dimensions of the measure μ and its support for systems satisfying some resonance conditions.

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Numerical study of a fixed point of Fibonacci renormalization

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(joint work with **Grzegorz Siudem**)

Following the results of [2] a numerical study was undertaken with the goal deciding whether Fibonacci circle coverings have a strange attractor. An important step in this project is to determine the fixed point of renormalization for a mapping with infinite criticality. The direction here is given by the work of [1] where a fixed point of renormalization was obtained for Feigenbaum maps with infinite criticality. Following the work of Epstein, the problem is first reduced to studying a map with a parabolic fixed point and its Fatou coordinate. While the algorithm similar to that used in [1] converges nicely in experiments, an analysis of its convergence remains work in progress. The arguments of [1] are extremely involved, which is in part because its authors set out an ambitious goal of verifying everything that the computer does. The talk will include a discussion of what might be an appropriate standard for mathematical analytis of numerical studies, taking into account mathematical rigor, credibility and comprehensibility.

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Acknowledgements
Both authors are supported in part by a grant UMO-2018/29/B/ST1/00013 funded by Narodowe Centrum Nauki.

Renormalisation theory for unimodal maps with strong asymmetries

Sebastian van Strien
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(joint work with **Oleg Kozlovski**)

Consider a family of unimodal maps with one linear and one quadratic branch. It turns out that surprisingly little is known about the dynamics of maps from such families. In this talk I will discuss renormalisation results in this setting.

On the rotation sets for maps of the torus T^d

Paulo Varandas
Federal University of Bahia, Brazil & University of Porto, Portugal
(joint work with **Heides Lima**)

In the attempt to generalize rotation theory for homeomorphisms on the torus, rotation sets were defined and soon became an important topic of research in low dimensional dynamics after pioneering works of Franks-Misiurewicz [1], Llibre-Mackay [2] and Misiurewicz-Ziemian [4, 5]. This important line of research in low dimensional dynamics have been intensively studied in the last few years, and many contributions address the stability of rotation sets and consequences of the shape of rotation sets. Most results deal with homeomorphisms on $\mathbb{T}^2$, because the methods of proofs depend on the convexity of rotations sets, which may fail in dimension $d > 2$.

In this talk I will consider homeomorphisms homotopic to the identity on the torus $\mathbb{T}^d$ ($d \geq 2$) and bring a different approach, using specification-like properties, to characterize the set of points with non-trivial rotation set (even being largest possible, which we denote by wild) and to show that "most" rotation sets are convex in any dimension $d \geq 2$. More precisely, (i) there exists a Baire residual subset of the set $\mathrm{Homeo}_{0,\lambda}(\mathbb{T}^2)$ so that the set of points with wild pointwise rotation set is a Baire residual subset in $\mathbb{T}^2$, and that it carries full topological pressure and full metric mean dimension, and (ii) for every $d \geq 2$ there exists a Baire residual subset of the set $\mathrm{Homeo}_{0,\lambda}(\mathbb{T}^d)$ formed by homeomorphisms on $\mathbb{T}^d$ with convex rotation sets. Related results are also obtained for dissipative homeomorphisms on tori.

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On dynamics of graph maps with zero topological entropy

Yini Yang
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(joint work with **Jian Li, Piotr Oprocha** and **Tiaoying Zeng**)

We explore the dynamics of graph maps with zero topological entropy. It is shown that a continuous map f on a topological graph G has zero topological entropy if and only if it is locally mean equicontinuous, that is the dynamics on each orbit closure is mean equicontinuous. As an application, we show that Sarnak's Möbius disjointness conjecture is true for graph maps with zero topological entropy. We also extend several results known in interval dynamics to graph maps. We show that a graph map has zero topological entropy if and only if there is no 3-scrambled tuple if and only if the proximal relation is an equivalence relation; a graph map has no scrambled pairs if and only if it is null if and only if it is tame.